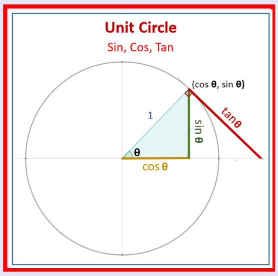
Math 241

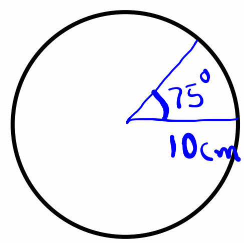
Winter 2024

Lecture 14



Feb 19-8:47 AM

Consider a Central angle on a Circle with radius 10cm that measures 75°.



1) Find its arc length

$$S = r\theta = \cancel{10}^{5} \cdot \frac{5\pi}{\cancel{12}_{6}} = \boxed{\frac{25\pi}{6}\text{ cm}}$$

$$75^\circ = 45^\circ + 30^\circ$$

$$= \frac{\pi}{4} + \frac{\pi}{6}$$

$$= \frac{3\pi + 2\pi}{12} = \boxed{\frac{5\pi}{12}}$$

2) Find its area.

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} \cdot 10^2 \cdot \frac{5\pi}{12} = \frac{\cancel{10}^{5} \cdot \cancel{10}^{5} \cdot 5\pi}{\cancel{2} \cdot \cancel{12}_{6}}$$

$$= \boxed{\frac{125\pi}{6}\text{ cm}^2}$$

Jan 25-8:02 AM

A Circular wheel makes 20 revolutions per min.
$r=15$

Its radius is 15cm.

$\omega = 20\cdot 2\pi = 40\pi$ Rad./min

Consider the point P on this wheel, find the linear speed of P.

v

$v = \frac{s}{t}$ $\quad \omega = \frac{\theta}{t}$ $\quad v = r\omega$

$v = 15 \cdot 40\pi$

$= \boxed{600\pi \text{ cm/min.}}$

Jan 25-8:09 AM

Convert 25.875° to DMS.

$25.875^\circ = 25^\circ \; .875(60)'$

$= 25^\circ \; 52.5'$

$= 25^\circ \; 52' \; .5(60)''$

$= 25^\circ \; 52' \; 30''$

Convert 25° 52' 30" to degrees.

$= 25 + \frac{52}{60} + \frac{30}{3600} = \boxed{25.875^\circ}$

Jan 25-8:16 AM

$m\angle A = 72^\circ\ 48'\ 50''$

$m\angle B = 17^\circ\ 55'\ 30''$

Find

1) $A + B = 89^\circ\ \underbrace{103'}_{60'+44'}\ \underbrace{80''}_{60'\ 20''}$

$= 89^\circ\ 104'\ 20''$

$= \boxed{90^\circ\ 44'\ 20''}$

2) $A - B$

$$\begin{array}{rrrr} & \cancel{72}^\circ\ {}^{71^\circ} & \cancel{48'}\ {}^{108'} & 50'' \\ - & 17^\circ & 55' & 30'' \\ \hline = & \boxed{54^\circ} & \boxed{53'} & \boxed{20''} \end{array}$$

Jan 25-8:26 AM

In triangle ABC, $a = 6cm$, $b = 10cm$, and $c = 12cm$. Find angle C.

Using Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$= \dfrac{6^2+10^2-12^2}{2(6)(10)}$

$= \dfrac{36+100-144}{120}$

$\cos C = \dfrac{\cancel{-8}^{\cancel{-2}^{-1}}}{\cancel{120}_{\cancel{30}_{15}}}$

$\cos C = \dfrac{-1}{15}$

$C = \cos^{-1}\left(\dfrac{-1}{15}\right) \approx \boxed{93.823^\circ}$

Jan 25-8:33 AM

Solve

$2A + 3 = 4$ $\quad 2A = 4 - 3$

$2A = 1 \rightarrow A = \frac{1}{2}$

$\left\{\frac{1}{2}\right\}$

Solve $2\cos x + 3 = 4$ on $[0, 2\pi)$

$2\cos x = 4 - 3$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

QI Angle = Ref. Angle

QIV Angle = 2π − Ref. Ang.

$x = \frac{\pi}{3}$

$x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Ref. angle $60° = \frac{\pi}{3}$

$\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

on $[0°, 360°) \rightarrow \{60°, 300°\}$

General Solutions

$\frac{\pi}{3} + 2n\pi$, $\frac{5\pi}{3} + 2n\pi$

Period for Sin & Cos is 2π.

Jan 25-8:42 AM

Solve $\sqrt{2}A - 6 = -7$

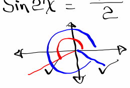
$\left\{-\frac{\sqrt{2}}{2}\right\}$

$\sqrt{2}A = -7 + 6$

$\sqrt{2}A = -1$ $\quad A = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ $\quad \boxed{A = -\frac{\sqrt{2}}{2}}$

Solve $\sqrt{2}\sin 2x - 6 = -7$

$\sin 2x = -\frac{\sqrt{2}}{2}$



Ref. angle $45° = \frac{\pi}{4}$

QIII $2x = \pi + \frac{\pi}{4} + 2n\pi$

General Solution

QIV $2x = 2\pi - \frac{\pi}{4} + 2n\pi$

$2x = \frac{5\pi}{4} + 2n\pi$ $\quad 2x = \frac{7\pi}{4} + 2n\pi$

Divide by 2 $\quad$ Divide by 2

$\boxed{x = \frac{5\pi}{8} + n\pi}$ $\quad \boxed{x = \frac{7\pi}{8} + n\pi}$

what about solutions $[0, 2\pi)$?

$n = 0 \rightarrow \frac{5\pi}{8}, \frac{7\pi}{8}$

$\left\{\frac{5\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

$n = 1 \rightarrow \frac{13\pi}{8}, \frac{15\pi}{8}$

$n = 2 \rightarrow$ No Answers

$\frac{5\pi}{8} + 2\pi$, $\frac{7\pi}{8} + 2\pi$

Jan 25-8:50 AM

Solve $A^2 - 1 = 0$

$(A+1)(A-1)=0$

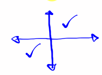
$A+1=0$ OR $A-1=0$

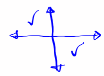
$A=-1$ $\quad$ $A=1$ $\quad$ $\{\pm 1\}$

$A^2 = 1 \to A = \pm\sqrt{1} = \pm 1$

Solve $\tan^2 x - 1 = 0$ on $[0, 2\pi)$

$\tan x = 1$ $\quad$ $\tan x = -1$





Ref. Angle $\frac{\pi}{4}$

Period for tan is π

QI Angle = Ref. Angle $+ n\pi$

QII Angle $= \pi -$ Ref. Angle $+ n\pi$

QIII Angle $= \pi +$ Ref. Angle $+ n\pi$

QIV Angle $= 2\pi -$ Ref. Angle $+ n\pi$

$x = \frac{\pi}{4} + n\pi$ QI

$x = \pi - \frac{\pi}{4} + n\pi$ QII

$x = \pi + \frac{\pi}{4} + n\pi$ QIII

$x = 2\pi - \frac{\pi}{4} + n\pi$ QIV

$$\boxed{\begin{aligned} x &= \frac{\pi}{4} + n\pi \\ x &= \frac{3\pi}{4} + n\pi \\ x &= \frac{5\pi}{4} + n\pi \\ x &= \frac{7\pi}{4} + n\pi \end{aligned}}$$

General Solutions

over $[0, 2\pi)$ $\to \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

$n=0 \to \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$n=1 \to \cancel{\frac{5\pi}{4}}, \cancel{\frac{7\pi}{4}}, \frac{9\pi}{4}, \frac{11\pi}{4}$ over $[0, 2\pi)$

$\frac{8\pi}{4} + \frac{\pi}{4} = 2\pi + \frac{\pi}{4}$

Jan 25-9:04 AM

Solve $2A^2 - A - 1 = 0$ by factoring method.

$(2A+1)(A-1)=0$

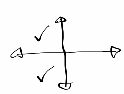
$2A+1=0$ OR $A-1=0$

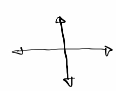
$A = -\frac{1}{2}$ $\quad$ $A = 1$ $\quad$ $\left\{-\frac{1}{2}, 1\right\}$

Solve $2\cos^2 \frac{1}{2}x - \cos\frac{1}{2}x - 1 = 0$

Factor $\left(2\cos\frac{1}{2}x + 1\right)\left(\cos\frac{1}{2}x - 1\right) = 0$

$\cos\frac{1}{2}x = -\frac{1}{2}$ $\quad$ $\cos\frac{1}{2}x = 1$





RA $= 60°$ $\quad$ RA $0°$

QII Angle $= 180° - 60° + n\cdot 360°$ $\quad$ Angle $= 0° + n\cdot 360°$

QIII Angle $= 180° + 60° + n\cdot 360°$

$\frac{1}{2}x = n\cdot 360°$

$\frac{1}{2}x = 120° + n\cdot 360° \to x = 240° + n\cdot 720°$

$x = n\cdot 720°$

$\frac{1}{2}x = 240° + n\cdot 360° \to x = 480° + n\cdot 720°$

$n=0 \to 240°, \cancel{480°}, 0° \to \{0°, 240°\}$

outside of $[0°, 360°)$

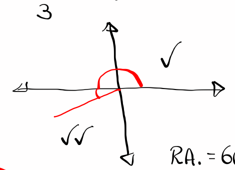
Jan 25-9:19 AM

Solve $\sqrt{3}\tan 2x = 3$

$\tan 2x = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

Period for tan.

$\tan 2x = \sqrt{3}$



R.A. $= 60° = \frac{\pi}{3}$

QI Angle = R.A. $+ n\pi$ → $2x = \frac{\pi}{3} + n\pi$

QIII Angle $= \pi + $ RA $+ n\pi$ → $2x = \pi + \frac{\pi}{3} + n\pi$

$\frac{4\pi}{3}$

$$\boxed{\begin{aligned} x &= \frac{\pi}{6} + \frac{n\pi}{2} \\ x &= \frac{2\pi}{3} + \frac{n\pi}{2} \end{aligned}}$$

$n = 0 \to \frac{\pi}{6}, \frac{2\pi}{3}$

$n = 1 \to \cancel{\frac{2\pi}{3}}, \frac{7\pi}{6}$

$\left\{\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}\right\}$

$\frac{\pi}{6} + \frac{\pi}{2} = \frac{4\pi}{6} = \frac{2\pi}{3}$, $\frac{2\pi}{3} + \frac{\pi}{2} = \frac{7\pi}{6}$

$n = 2 \to \cancel{\frac{7\pi}{6}}, \frac{5\pi}{3}$

$n = 3 \to$

$\frac{\pi}{6} + \frac{2\pi}{2} = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$

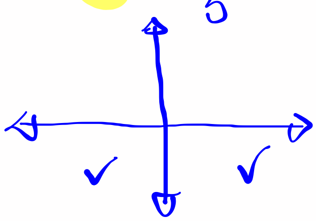
$\frac{\pi}{6} + \frac{3\pi}{2} = \frac{10\pi}{6} = \cancel{\frac{5\pi}{3}}$

$\frac{2\pi}{3} + \frac{2\pi}{2} = \frac{2\pi}{3} + \pi = \frac{5\pi}{3}$

Jan 25-10:00 AM

Solve $5\sin x + 3 = 0$ over $[0°, 360°)$.

$\sin x = \frac{-3}{5}$



R.A. $= \sin^{-1}\left(\frac{3}{5}\right) \approx 37°$

QIII Angle $= 180° + $ R.A. $+ n \cdot 360° = 180° + 37° + n \cdot 360°$

QIV Angle $= 360° - $ RA $+ n \cdot 360° = 360° - 37° + n \cdot 360°$

$$\boxed{\begin{aligned} x &= 217° + n \cdot 360° \\ x &= 323° + n \cdot 360° \end{aligned}}$$

$n = 0$

$x = 217°, 323°$

Jan 25-10:11 AM

Solve

$\sin x - \sqrt{3}\cos x = 0$

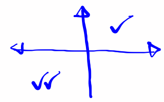
Divide by $\cos x$, and Simplify

$$\frac{\sin x}{\cos x} - \frac{\sqrt{3}\,\cancel{\cos x}}{\cancel{\cos x}} = \frac{0}{\cos x}$$

$\tan x - \sqrt{3} = 0$

Isolate $\tan x$, identify quadrant, find RA.

$\tan x = \sqrt{3}$



QI Angle = RA + n · Period

$RA = \frac{\pi}{3} = 60^\circ$

QIII Angle = 180° + RA + n · Period

$x = 60^\circ + n \cdot 180^\circ$

$x = 180^\circ + 60^\circ + n \cdot 180^\circ$

$\Rightarrow$ $\boxed{x = 60^\circ + n \cdot 180^\circ}$ $\boxed{x = 240^\circ + n \cdot 180^\circ}$

$n = 0 \rightarrow 60^\circ, 240^\circ$

$n = 1 \rightarrow \cancel{240^\circ}, \cancel{420^\circ}$

$\Rightarrow \{60^\circ, 240^\circ\}$

Jan 25-10:17 AM

Solve $\boxed{4 \sin x \cos x} = \sqrt{3}$ Give all exact solutions in $\boxed{\text{Radians}}$.

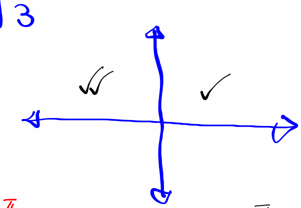
Recall $\sin 2x = 2 \sin x \cos x$

$2 \cdot 2 \sin x \cos x = \sqrt{3}$

$2 \cdot \sin 2x = \sqrt{3}$

$\sin 2x = \frac{\sqrt{3}}{2}$

2π



$RA = 60^\circ = \frac{\pi}{3}$

QI Angle = RA + n · period

QII Angle = π − RA + n · Period

QI $2x = \frac{\pi}{3} + n \cdot 2\pi \rightarrow$ $\boxed{x = \frac{\pi}{6} + n\pi}$

QII $2x = \left(\pi - \frac{\pi}{3}\right) + n \cdot 2\pi$ ($2\pi/3$) $\rightarrow$ $\boxed{x = \frac{\pi}{3} + n\pi}$

Jan 25-10:27 AM

Find all Solutions over $[0°, 360°)$ for

$\sin 2x = \cos 2x + 1$.

Hint: Move $\cos 2x$ to the left, then Square both Sides.

$(A-B)^2 = A^2 - 2AB + B^2$

$(\sin 2x - \cos 2x)^2 = 1^2$

$\sin^2 2x - 2\sin 2x \cos 2x + \cos^2 2x = 1$

$\cancel{1} - 2\sin 2x \cos 2x = \cancel{1}$

$-\sin 2(2x) = 0$

$\sin 4x = 0$ $0°$ & $180°$

Angle $= 0° + n \cdot$ Period

Angle $= 180° + n \cdot$ Period

$4x = 0° + n \cdot 360° \rightarrow x = n \cdot 90°$

$4x = 180° + n \cdot 360° \rightarrow x = 45° + n \cdot 90°$

$n=0 \rightarrow \cancel{0}, \boxed{45°}$ $n=2 \rightarrow 180°, 225°$

$n=1 \rightarrow 90°, 135°$ $n=3 \rightarrow 270°, 315°$

Check $x = 0°$

$\sin 2x = \cos 2x + 1$

$\sin 0° = \cos 0° + 1$

$0 = 1 + 1$

$0 \neq 2$

$x = 45°$

$\sin 2x = \cos 2x + 1$

$\sin 90° = \cos 90° + 1$

$1 = 0 + 1$

$1 = 1$ ✓

Anytime You square both Sides, You must check ans for Possible extraneous Soln.

Suppose

$\sqrt{A} = -4$ No Soln.

but If we square both Sides

$(\sqrt{A})^2 = (-4)^2 \rightarrow \boxed{A = 16}$

$\rightarrow \sqrt{16} = -4$

$4 = -4$

Jan 25-10:36 AM

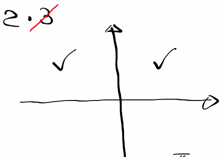
Solve

$2\sqrt{3}\sin\frac{x}{2} - 3 = 0$

Find all exact Solutions over $[0, 2\pi)$

$\sin\frac{x}{2} = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{2 \cdot \cancel{3}}$

$\sin\frac{x}{2} = \frac{\sqrt{3}}{2}$



$RA = \frac{\pi}{3}$

QI   Angle $= RA + n \cdot$ Period

QII   Angle $= \pi - RA + n \cdot$ Period

$\frac{x}{2} = \frac{\pi}{3} + n \cdot 2\pi \rightarrow x = \frac{2\pi}{3} + 4n\pi$

$\frac{x}{2} = \pi - \frac{\pi}{3} + n \cdot 2\pi \rightarrow x = \frac{4\pi}{3} + 4n\pi$

$n = 0 \rightarrow \frac{2\pi}{3}, \frac{4\pi}{3} \Longrightarrow \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

$n = 1 \rightarrow$ Outside of $[0, 2\pi)$ Interval

Jan 25-10:56 AM

Solve all exact Solutions in radians for

$$\csc x - \cot x = 1$$

Recall

$$1 + \cot^2 x = \csc^2 x$$

Let's square both Sides

$$[\csc x - \cot x]^2 = 1^2$$

$$\csc^2 x - 2\csc x \cot x + \cot^2 x = 1$$

$$\cancel{1} + \cot^2 x - 2\csc x \cot x + \cot^2 x = \cancel{1}$$

$$2\cot^2 x - 2\csc x \cot x = 0$$

$$2\cot x\,[\cot x - \csc x] = 0$$

$\cot x = 0$

$90^\circ, 270^\circ$

$\cot x - \csc x = 0$

$$\frac{\cos x}{\sin x} - \frac{1}{\sin x} = 0$$

LCD = $\sin x$

$\cos x - 1 = 0$

$\cos x = 1$

0°

$x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$

Since we Square both Sides, we must check all Solutions

$\cancel{x=0}$ $\quad \cancel{\csc 0} - \cot 0 = 1$

only Solution. $\rightarrow$ $\boxed{x = \frac{\pi}{2}}$ $\quad \csc\frac{\pi}{2}^{1} - \cot\frac{\pi}{2}^{0} = 1$ ✓

$\cancel{x = \frac{3\pi}{2}}$ $\quad \csc\frac{3\pi}{2}^{-1} - \cot\frac{3\pi}{2}^{0} = 1$ $\quad -1 \neq 1$

$$\boxed{\pi/2 + 2n\pi}$$

Jan 25-11:08 AM